MEASURING VOTING POWER USING OBSERVED DATA

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Abstract. The Shapley-Shubik and Banzhaf power indices have been used to measure the voting power of voters in a variety of political institutions, including the United Nations Security Council, the United States electoral college, and proportionate representation legislatures in a variety of countries. These power indices depend only upon the voting rules; hence, each assigns an equal power to each of the nine Justices on the United States Supreme Court, because each Justice has one vote and cases are determined by simple majority. Once actual people having particular ideological positions and behavioral patterns become members of a political institution, their true voting power can be measured more accurately by taking into account their past behavior and political alignment with respect to other voters; for example, the Justice seen as occupying the central position in the liberal to conservative ordering of U.S. Supreme Court Justices is often considered more powerful than the other Justices. This paper examines power indices that take into account both the voting rules and the voting records of voters and applies these indices to the United States Senate.

1. Introduction

In a weighted voting game, the Shapley-Shubik power index examines each possible ordering of the players and finds a pivotal voter in each ordering, taking the average as the player's relative power. Meanwhile, the Banzhaf power index examines each possible coalition of players, marking all pivotal voters in each coalition and using that average as the player's relative power. The goal of this paper, similar to that of Frank and Shapley and Edelman and Chen is to extend these power indices to also take into account the past voting history of each player, to gain a more accurate sense of relative power in particular political institutions with particular voters. The resulting power index is applied to the United States Senate using 32 substantive votes that occurred in 2017.

2. Definitions

To find the power of voters in the Senate, the idea of a weighted voting game (Definition 3) is used to describe the voting system itself. This is a special type of coalition game (Definition 1) in which the outcome is binary.

Definition 1. *A* ***coalition game*** (N, w) *consists of the following:*

(1) *A set* $N = \{1, 2, ..., n\}$ *of at least two players along with all nonempty subsets* $S \subseteq N$*, called coalitions.*
(2) *For each coalition* S*, a determined worth* $w(S) \subseteq \mathbb{R}$*.*
(3) *Utilities* u_i *given by the payoff received by player* i*.*

Definition 2. *A* ***voting game*** *is a coalition game* (N, w) *for which:*

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(1) *$w(S) = 1$ or $w(S) = 0$ for each coalition S.*
(2) *$w(N) = 1$.*
(3) *$w(T) = 1$ whenever $S \subset T$ and $w(S) = 1$.*
(4) *$w(N \backslash S) = 0$ whenever $w(S) = 1$.*

The coalitions S satisfying $w(S) = 1$ are called winning, and the coalitions S satisfying $w(S) = 0$ are called losing. Let S_c denote the set of all coalitions.

Definition 3. *A **weighted voting system**, denoted by $[q; v_1, v_2, ..., v_n]$, consists of a positive quota q and nonnegative voting weights $v_1, v_2, ..., v_n$ for the n voters. The weighted voting system $[q; v_1, v_2, ..., v_n]$ is called a representation of the n-player **weighted voting game** for which coalition S is winning if and only if $\sum_{i \in S} v_i \geq q$.*

In a weighted voting game, the Shapley-Shubik power index can be used to measure each player's relative power. This method considers all possible orderings of players as being equally likely, finding the 'pivotal' player in each ordering who could swing the election.

Definition 4. *For a weighted voting game represented by $[q; v_1, v_2, ..., v_n]$, let $O = \{p_1, p_2, ..., p_{n!}\}$ be the set of all permutations of N. For each ordering $p_k \in O$, let $M_j(p_k)$ be the set of all players in the ordering up to and including player $j \in N$. Then for each $p_k \in O$, the **marginal contribution** of player i is given by*

$$c_i(p_k) = \begin{cases} 1 & if\, w(M_i(p_k)) \geq q \text{ and } w(M_i(p_k) \backslash i) < q \\ 0 & otherwise \end{cases}$$

*If $c_i(p_k) = 1$, we say that player i is **pivotal** on the ordering p_k. The **Shapley-Shubik power** of player $i \in N$ is given by*

$$\varphi(i) = \frac{\sum\limits_{p_k \in O} c_i(p_k)}{\sum\limits_{j \in N} \sum\limits_{p_k \in O} c_j(p_k)}$$

Rather than orderings, the Banzhaf power index considers all possible coalitions of players as being equally likely. In each winning coalition, if there are any players that could unilaterally drop out of the coalition and cause it to become losing by doing so, each is marked as pivotal.

Definition 5. *For a weighted voting game represented by $[q; v_1, v_2, ..., v_n]$, let $\mathcal{S}_i$ denote the set of all winning coalitions S containing player i for which $S \backslash i$ is losing; in each of these, player i is considered **pivotal**. Then the **Banzhaf power** of player i is given by*

$$\beta(i) = \frac{|\mathcal{S}_i|}{\sum\limits_{j \in N} |\mathcal{S}_j|}$$

These two power indices are commonly used to find the *a priori* power of voters in a voting system. However, we are interested in using real voting data from particular voting systems to better capture the actual power of each voter.

3. Transforming Voting Data Into an Issue Space

3.1. **Voting Data.** Consider a weighted voting game represented by $[q; v_1, v_2, ..., v_n]$. Suppose the players have voted on multiple cases in the past (let C be the set of all such cases) and their votes have been recorded in a vote matrix:

Definition 6. *A* ***vote matrix*** *for a weighted voting game is any matrix X of the following form:*

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{c1} & x_{c2} & x_{c3} & \dots & x_{cn} \end{bmatrix}$$

where $x_{ij} = 1$ if player $i \in N$ voted Yea on issue $j \in C$, $x_{ij} = -1$ if player i voted Nay on issue j, and $x_{ij} = 0$ if player i abstained from voting on issue j.

Each row of X describes how each player voted on a particular case; each column describes how a particular player voted on each case.

3.2. **Issue Space.** In order to more accurately assess the relative power of each voter in the above case, we would like to plot each voter as a point in some issue space $\mathbb{R}^m$ so that we may use the techniques explained in Section 4. The most obvious way to do this is to simply plot each column of X as a point in $\mathbb{R}^c$. However, if n and c are relatively large - as they are in the case of the U.S. Senate - it becomes computationally infeasible to calculate the techniques of Section 4 in this way. Therefore, the dimension of the issue space must be reduced.

One way of doing this is to consider a real-life phenomenon. A number of political interest groups regularly rate each Congressperson based on their voting history, assigning them a percentage score that describes how well the Congressperson's ideology aligns with that of the interest group. Presumably, this is done in a similar manner to the following: for each issue that comes before Congress, the interest group decides whether it wants the issue to pass or fail, and how strongly they prefer that outcome. This can be represented by a real number between 1 and -1, with a 1 indicating a strong preference for passage, a -1 indicating a strong preference for failure, and a 0 indicating indifference.

Definition 7. *A* ***weight vector*** *for a vote matrix X is a vector $y = [y_1, y_2, ..., y_c]$ where $y_j \in \mathbb{R}$ is in the closed interval $[-1, 1]$ for all $j \in C$.*

To evaluate Congress (or the set of voters in any weighted voting game) on a set of issues, the interest group can simply decide on its weight vector y and take the matrix product yX to obtain a vector z, each element z_i of which indicates the interest group's overall level of agreement with voter i. 1 indicates strong agreement, -1 indicates strong disagreement, and 0 could indicate either indifference or mixed feelings.

Definition 8. *The* ***evaluation vector*** *z for a vote matrix X and a weight vector y is the product $z = yX$ in $\mathbb{R}^c$.*

In this way, the interest group has essentially reduced the issue space down to a single dimension which corresponds to the interest group's ideology. Of course, this is an incomplete representation of the situation; after all, Planned Parenthood is unlikely to care about foreign policy issues and will not factor them into its evaluation of Congress. However, by using multiple interest groups, the dimension of the issue space could be reduced while still giving a more complex representation of the situation.

3.3. **Random Generation of Interest Groups.** Due to inconsistent data, as well as the subjectivity that comes from choosing particular real interest groups to represent important issues, actual interest group data was not used in this project. However, a pseudo-random number generator was used to uniformly generate numbers between -1 and 1 to serve as elements of weight vectors. These random weight vectors were then used to reduce the dimensionality of the issue space.

Definition 9. *A **weight matrix** A for a vote matrix X is any matrix of the form:*

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1c} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2c} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{d1} & a_{d2} & a_{d3} & \dots & a_{dc} \end{bmatrix}$$

where each row of A is a weight vector.

Definition 10. *The **evaluation matrix** Z for a vote matrix X and a weight matrix A is the matrix product AX. Each row of Z is an evaluation vector.*

Therefore, to reduce the dimensionality of the issue space $\mathbb{R}^c$, we can randomly generate a weight matrix A with a chosen number m of rows, multiply by the vote matrix X, and plot each column of the resulting evaluation matrix Z as a point in $\mathbb{R}^m$.

4. Finding the Power of Voters in an Issue Space

Once voters have been placed in a suitable issue space, we can attempt to find their relative power by extending the ideas of the Shapley-Shubik and Banzhaf power indices into the issue space representation.

4.1. **Extending the Shapley-Shubik Power Index.** The Shapley-Shubik power index considers each ordering of players as being equally likely. However, perhaps some orderings are more or less likely than others. We use the method described by Frank and Shapley to effectively attach weights to each ordering describing the likelihood of the ordering.

Imagine a hypothetical issue (one on which the voters could, in theory, hold a vote) as a direction in the issue space. Now consider a hyperplane perpendicular to the issue direction, positioned away from the direction so that all voters are on one side of it. Now imagine sweeping that hyperplane through the voters in the chosen direction, passing through them in a particular order. We can now proceed as we would with the original Shapley-Shubik index: go through the voters in order, add up their votes, and label as pivotal the voter that causes the vote sum to exceed the quota.

If we were to do this for all possible issues (all possible directions in the issue space), we could map each player to a region on the unit hypersphere corresponding to the directions in which that player is the pivotal voter. Then each player's relative power would be given by the proportion of the surface area of the unit hypersphere to which they are mapped.

Definition 11. *Let* $[q; v_1, v_2, ..., v_n]$ *be a weighted voting game with evaluation matrix* Z*. Define the set of unit vectors* $\Delta = \{b : b \in \mathbb{R}^m$ *and* $|b| = 1\}$*, and let* B *be a random vector uniformly distributed over* Δ*. To order the players on a direction, define the function* $T : B \to$ *permutation of* N*, which considers each element* $p_i \in BZ$*, creates a list* R *of all the elements of* BZ *sorted in ascending order, and finally replaces each element* $p_i \in R$ *with* i*, returning* $T(B) = R$*. Then the* ***Extended Shapley-Shubik power*** *of player* $i \in N$ *is given by*

$$\varphi(i) = E[c_i(T(B))]$$

where E *is the expectation operator, and* c *is defined as in Definition 4.*

4.2. Extending the Banzhaf Power Index.

The Banzhaf power index considers each coalition of players as being equally likely. However, perhaps some coalitions will realistically never form due to the ideologies of the voters (the geometry of the points in the issue space).

Imagine a coalition as a hypersphere in $\mathbb{R}^m$ that contains all the points corresponding to the players in the coalition, and contains no points corresponding to the players outside of the coalition. Then some coalitions will not be possible; that is, there will not exist a hypersphere that contains all points in the coalition without containing points outside of the coalition. If we proceed as we would for the simple Banzhaf power index but only consider coalitions that are possible, we may have a more accurate sense of the voters' power.

Definition 12. *Let* $V = [q; v_1, v_2, ..., v_n]$ *be a weighted voting game with vote matrix* X *and evaluation matrix* $Z = [z_1, z_2, ..., z_n]$*, and let* $S \subseteq N$ *be a coalition. Then* S *is* ***possible*** *if there exists a hypersphere* Q *in* $\mathbb{R}^m$ *such that*

$$t \in Q \quad \forall \quad t \in \{z_i \mid i \in S\}$$

$$u \notin Q \quad \forall \quad u \in \{z_i \mid i \notin S\}$$

Let $\mathcal{S}_i$ *denote the set of all possible winning coalitions* S *containing* i *for which* $S \backslash i$ *is possible and losing.*

Definition 13. *For a weighted voting game represented by* $[q; v_1, v_2, ..., v_n]$*, the* ***Extended Banzhaf power*** *of player* $i \in N$ *is given by*

$$\beta(i) = \frac{|\mathcal{S}_i|}{\sum\limits_{j \in N} |\mathcal{S}_j|}$$

5. The United States Senate

The U.S. Senate can be thought of as the weighted voting game $[51; 1, 1, ..., 1]$ with a total of 101 1's after the 51. While there are technically only 100 Senators, the Vice President is allowed to cast a vote in the case of a tie; therefore, the Vice President has as much power as a Senator, since he or she is able to vote in every case where his or her vote would swing the election. The vote matrix used in this analysis consists of the 32 "Passage" votes conducted in the Senate in 2017, gathered from the website www.govtrack.us/congress/votes.

The Vice President only voted in the case of a tie, so a 0 (for Not Voting) was recorded in cases that were not tied. Additionally, Senator Jeff Sessions was promoted to Attorney General in early 2017 and subsequently replaced by Senator Luther Strange. Since Senator Sessions was present for only 5 of the 32 votes, he was removed from this analysis entirely, and in his place, a 0 was recorded for Senator Strange on those 5 cases.

Due to the computation time involved in calculating the Extended Shapley-Shubik and Extended Banzhaf power indices for 101 players in multiple dimensions, this analysis only considers a single dimension at a time (that is, the weight matrix consists of a single weight vector). This results in a single evaluation vector. However, the process of generating a weight vector and producing an evaluation vector was repeated 1000 times, and the resulting Extended Shapley-Shubik and Extended Banzhaf power indices were averaged separately. Then, that whole process was repeated 1000 times, with each set of averages stored as a datum, and the resulting data were used to create two box-and-whisker plots, one for each power index. These are displayed in the appendix. The x-axis contains the 101 senators, and the y-axis shows the spread and median of their power on a scale from 0 to 1.

A fair number of Senators voted in exactly the same way on all 32 issues as at least one other Senator; this explains why many of the boxes look identical. Additionally, one may notice that the Senators are in the same order in both graphs. This will always be the case in one dimension, due to the following result.

Theorem 1. *Let $V = [q; v_1, v_2, ..., v_n]$ be a weighted voting game with vote matrix X and evaluation matrix $Z = [z_1, z_2, ..., z_n]$, where $v_1 = v_2 = ... = v_n = 1$ and $q = \left\lfloor \frac{n}{2} \right\rfloor + 1$. If Z contains exactly one row, then the relative power orderings produced by the Extended Shapley-Shubik and Extended Banzhaf methods are the same. That is, for all $i, j \in N$, $\varphi(i) < \varphi(j)$ if and only if $\beta(i) < \beta(j)$.*

Proof. In one dimension, a hypersphere is simply a line segment. Thus, if coalition S includes the voter with point Z_i and the voter with point Z_j such that $Z_i \leq Z_j$, then S also includes all voters with points Z_k such that $Z_i \leq Z_k \leq Z_j$. Note also that since all players have 1 vote and the quota is a simple majority, all minimally winning coalitions contain a number of voters equal to the quota. Let $Q = [q_1, q_2, ..., q_n]$ be the list of players as ordered on the line; that is, $Z_{q_1} \leq Z_{q_2} \leq ... \leq Z_{q_n}$.

Then the possible minimally winning coalitions are

$$\{q_1, q_2, ..., q_l\}, \{q_2, q_3, ..., q_{l+1}\}, ..., \{q_l, q_{l+1}, ..., q_n\} \text{ where } l = \left\lfloor \frac{n}{2} \right\rfloor + 1$$

and furthermore, for each possible minimally winning coalition S containing player i, $S \backslash i$ is always losing, but is only possible if $Z_i \leq Z_j$ for all $j \in S$ or if $Z_i \geq Z_j$ for all $j \in S$; that is, only the endpoints of a coalition are pivotal, because no other member could leave without breaking the circle into two pieces. Observe that only q_l is an endpoint in two of the possible minimally winning coalitions; all other player points are endpoints in only one such coalition. Therefore, $\beta(i) = frac2n+1$ if $i = l$ and $\beta(i) = frac1n+1$ otherwise.

Suppose $\varphi(i) < \varphi(j)$; that is, that $E[c_i(T(B))] < E[c_j(T(B))]$. Then for some ordering $T(b)$ (call the direction 'left'), $c_i(T(b)) = 0$ but $c_j(T(b)) = 1$, which means that $w(M_j(T(b))) \geq q$ and $w(M_j(T(b))) < q$ but either $w(M_i(T(b))) < q$ or $w(M_i(T(b))) \geq q$. Suppose $w(M_i(T(b))) < q$; then fewer than half of the players come before player i in the ordering, which means that i can only be the left endpoint of a minimally winning coalition in the Banzhaf calculation, and since there are only two directions, i is Banzhaf pivotal in exactly one minimally winning coalition. Suppose $w(M_i(T(b))) \geq q$; then more than half of the players come before player i in the ordering, which means that i can only be the right endpoint of a minimally winning coalition in the Banzhaf calculation, and since there are only two directions, i is Banzhaf pivotal in exactly one minimally winning coalition. Since $E[c_i(T(B))] < E[c_j(T(B))]$ and there are only 2 possible orderings, it must not be the case that on the other ordering, $w(M_i(T(b))) \geq q$ and $w(M_i(T(b))) < q$ but either $w(M_j(T(b))) < q$ or $w(M_j(T(b))) \geq q$. Therefore, either $w(M_i(T(b))) < q$, $w(M_i(T(b))) \geq q$, or both $w(M_j(T(b))) \geq q$ and $w(M_j(T(b))) < q$. If the third is true, then exactly half of the players other than j are on j's left and half are on the right, which means j is Banzhaf pivotal twice, and thus $\beta(i) < \beta(j)$.

Suppose $\beta(i) < \beta(j)$; that is, $\frac{|\mathcal{S}_i|}{\sum\limits_{k \in N} |\mathcal{S}_k|} < \frac{|\mathcal{S}_j|}{\sum\limits_{k \in N} |\mathcal{S}_k|}$. Then player i must be Banzhaf pivotal in exactly one minimally winning coalition, and player j must be pivotal in exactly two. Therefore, exactly half of the players other than j must be to j's left, and half to j's right. This means that, for both possible orderings $T(b)$, $w(M_i(T(b))) = q$ and $w(M_j(T(b))) < q$. However, either more than half of the players other than i must be to i's right, or more than half of the players other than i must be to i's left. If the former, then $w(M_i(T(b))) \geq q$, and if the latter, then $w(M_i(T(b))) < q$, so either way, $\varphi(i) < \varphi(j)$.

□

The only difference in one dimension between these two power indices, then, is that the Extended Shapley-Shubik method awards more power to the most powerful voters and less power to the least powerful voters than does the Extended Banzhaf.

According to this analysis, the most powerful politician was Senator Johnny Isakson of Georgia, who was assigned a median of 0.043 by the Extended Shapley-Shubik index and a 0.0102 by the Extended Banzhaf index. The least powerful was a seven-way tie between Senators Jack Reed, Chuck Schumer, Sherrod Brown, Amy Klobuchar, Bob Casey Jr., Sheldon Whitehouse, and Brian Schatz, who each were assigned 0.005 by the Extended Shapley-Shubik and 0.0097 by the Extended Banzhaf.

Upon looking at the list of Senators in order of power (see the appendix), it becomes apparent that party alignment is significant. Every single one of the 35 least powerful Senators is a Democrat, while the top 16 are all Republican. This makes some sense, as the Republicans comfortably hold a simple majority in the Senate, which means that the most powerful Senators ought to be moderate Republicans, not Democrats. However, one might also expect some of the most far-right Republicans to be near the bottom of the ranking as well, which is not the case.

6. Acknowledgements

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7. Appendix

Each Senator was assigned an ID, which is displayed on the x-axis of the following box plots. It can also be seen next to their name in the following list.

List of Senators in ascending order of power:

20 Sen. John “Jack” Reed [D]
22 Sen. Charles “Chuck” Schumer [D]
29 Sen. Sherrod Brown [D]
50 Sen. Amy Klobuchar [D]
53 Sen. Robert “Bob” Casey Jr. [D]
54 Sen. Sheldon Whitehouse [D]
78 Sen. Brian Schatz [D]
1 Sen. Maria Cantwell [D]
26 Sen. Tammy Baldwin [D]
64 Sen. Michael Bennet [D]
65 Sen. Alan “Al” Franken [D]
80 Sen. Tammy Duckworth [D]
81 Sen. Elizabeth Warren [D]
18 Sen. Patty Murray [D]
42 Sen. Chris Van Hollen Jr. [D]
45 Sen. Christopher Murphy [D]
72 Sen. Richard Blumenthal [D]
2 Sen. Thomas Carper [D]
24 Sen. Debbie Stabenow [D]
58 Sen. Martin Heinrich [D]
59 Sen. Gary Peters [D]
60 Sen. Mark Warner [D]
62 Sen. Jeanne Shaheen [D]
66 Sen. Chris Coons [D]
87 Sen. Timothy Kaine [D]
97 Sen. Margaret “Maggie” Hassan [D]
9 Sen. Dianne Feinstein [D]
25 Sen. Ron Wyden [D]
32 Sen. Benjamin Cardin [D]
36 Sen. Robert “Bob” Menéndez [D]
49 Sen. Kirsten Gillibrand [D]
63 Sen. Jeff Merkley [D]
88 Sen. Cory Booker [D]
46 Sen. Mazie Hirono [D]
95 Sen. Kamala Harris [D]
3 Sen. Thad Cochran [R]
5 Sen. John Cornyn [R]
12 Sen. Orrin Hatch [R]
16 Sen. Mitch McConnell [R]
21 Sen. Pat Roberts [R]
27 Sen. Roy Blunt [R]
28 Sen. John Boozman [R]
31 Sen. Shelley Capito [R]
43 Sen. Roger Wicker [R]

44 Sen. John Thune [R]
57 Sen. Bill Cassidy [R]
68 Sen. Cory Gardner [R]
7 Sen. Richard Durbin [D]
14 Sen. Patrick Leahy [D]
35 Sen. Edward “Ed” Markey [D]
39 Sen. Bernard “Bernie” Sanders [I]
41 Sen. Tom Udall [D]
98 Sen. Catherine Cortez Masto [D]
11 Sen. Charles “Chuck” Grassley [R]
19 Sen. Bill Nelson [D]
61 Sen. James Risch [R]
73 Sen. Marco Rubio [R]
83 Sen. Steve Daines [R]
85 Sen. Deb Fischer [R]
48 Sen. Dean Heller [R]
71 Sen. Tim Scott [R]
6 Sen. Michael Crapo [R]
79 Sen. Tom Cotton [R]
30 Sen. Richard Burr [R]
75 Sen. John Hoeven [R]
8 Sen. Michael Enzi [R]
23 Sen. Richard Shelby [R]
56 Sen. John Barrasso [R]
70 Sen. James Lankford [R]
77 Sen. Ron Johnson [R]
90 Sen. David Perdue [R]
0 Sen. Lamar Alexander [R]
40 Sen. Patrick “Pat” Toomey [R]
51 Sen. Claire McCaskill [D]
52 Sen. Jon Tester [D]
92 Sen. Thom Tillis [R]
13 Sen. James “Jim” Inhofe [R]
67 Sen. Joe Manchin III [D]
91 Sen. Joni Ernst [R]
94 Sen. Benjamin Sasse [R]
47 Sen. Joe Donnelly [D]
84 Sen. Heidi Heitkamp [D]
33 Sen. Jeff Flake [R]
37 Sen. Jerry Moran [R]
82 Sen. Angus King [I]
86 Sen. Ted Cruz [R]
89 Sen. Dan Sullivan [R]
93 Sen. Mike Rounds [R]
76 Sen. Mike Lee [R]
10 Sen. Lindsey Graham [R]
96 Sen. John Kennedy [R]
15 Sen. John McCain [R]

69 Sen. Todd Young [R]
38 Sen. Robert "Rob" Portman [R]
99 Sen. Luther Strange [R]
17 Sen. Lisa Murkowski [R]
74 Sen. Rand Paul [R]
4 Sen. Susan Collins [R]
55 Sen. Bob Corker [R]
100 Vice President Mike Pence [R]
34 Sen. John "Johnny" Isakson [R]

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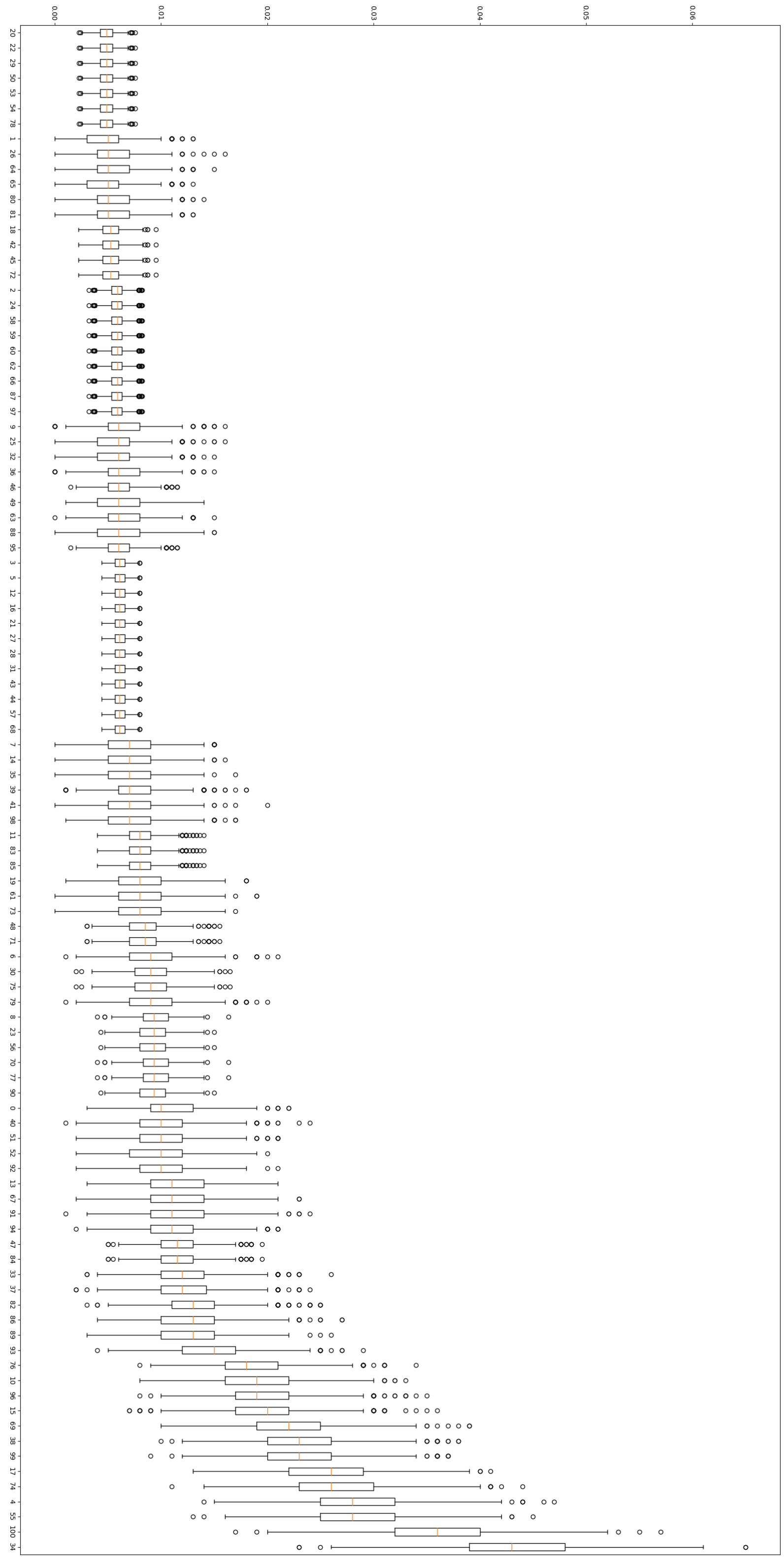

FIGURE 1. Box-and-whisker plot for the Senate using the Extended Shapley-Shubik power index

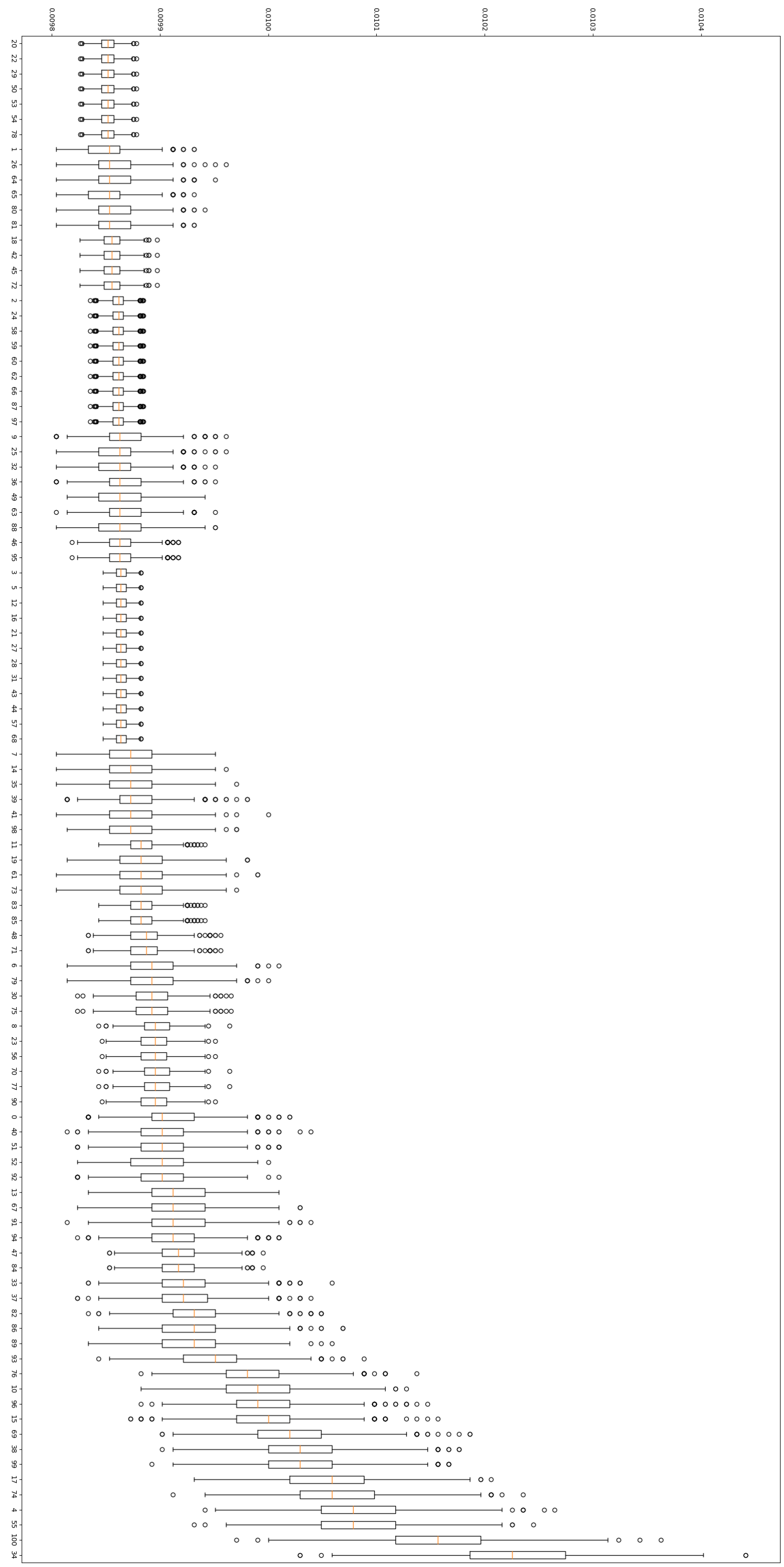

FIGURE 2. Box-and-whisker plot for the Senate using the Extended Banzhaf power index